SOFT GLUON RESUMMATION
vs
PARTON SHOWER SIMULATIONS

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We limit ourselves to considering hard scattering events: the starting point is the factorization theorem

\[ \sigma(p_1, p_2; Q^2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_{h_1,a}(x_1, \mu_F^2) f_{h_2,b}(x_2, \mu_F^2) \times \hat{\sigma}_{ab}(x_1 p_1, x_2 p_2, \alpha_S(Q^2), \mu_F^2) \]

Predictions for the hard cross section can be obtained through:

- Fixed order calculations
- All-order resummed calculations
- Parton Shower simulations
Fixed order calculations

Truncate perturbative expansion at a given order in $\alpha_s$

$$\hat{\sigma}(p_1, p_2; Q\{Q_1, \ldots\}; \mu_F) = \alpha_s^k(\mu_R)\left(\hat{\sigma}^{(LO)}(p_1, p_2; Q\{Q_1, \ldots\}) + \alpha_s(\mu_R)\hat{\sigma}^{(NLO)}(p_1, p_2; Q\{Q_1, \ldots\}; \mu_F, \mu_R) + \alpha_s^2(\mu_R)\hat{\sigma}^{(NNLO)}(p_1, p_2; Q\{Q_1, \ldots\}; \mu_F, \mu_R) + \ldots\right)$$

It provides a systematic framework to compute the partonic cross section for an inclusive enough hard scattering process

It is reliable only when all the scales are of the same order: if $Q_1 \gg Q$ large logarithmic contributions of the form

$$(\alpha_s L^2)^n$$ with $L \equiv \ln Q_1/Q$ arise that may spoil the perturbative expansion
Soft-gluon resummations

In general: even if KLN theorem guarantees the cancellation of IR singularities, soft-gluon effects can still be large when real and virtual contributions are kinematically unbalanced.

\[ \alpha_s L^2 \]: one soft and one collinear log for each power of \( \alpha_s \)

\[ \alpha_s L \]

Look for an improved perturbative expansion when \( \alpha_s L \sim 1 \)

\[ \alpha_s L^2 \quad \alpha_s^2 L^4 \quad \ldots \quad \alpha_s^n L^{2n} \quad \text{Leading Logs (LL)} \]

\[ \alpha_s L \quad \alpha_s^2 L^3 \quad \ldots \quad \alpha_s^n L^{2n-1} \quad \text{Next to Leading Logs (NLL)} \]
Resummation is possible if the observable fulfills the property known as **exponentiation**

This implies two basic conditions:

- Matrix element factorization
- Phase space factorization

The first is a consequence of **gauge invariance** and **unitarity**: in soft and collinear limits the singular structure of QCD matrix elements can be factored out in a (universal) process independent manner.

The second condition regards **kinematics** and depends on the cross section considered. If this condition can be fulfilled (typically working in a conjugate space), resummation is feasible.
Well known examples:

- Event shapes and jet rates in $e^+e^-$
- Resummation of soft gluons near threshold $z = Q^2/\hat{s} \to 1$
- $q_T$ distributions in hadron collisions or Energy-Energy Correlation in $e^+e^-$

Typically use Laplace transform
General structure of resummed cross section \( \nu \): 

\[
\hat{\sigma}_\nu^{\text{res.}} \sim \alpha_S^k \hat{\sigma}_\nu^{(LO)} H(\alpha_S) \exp \{ L g_1(\alpha_S L_\nu) + g_2(\alpha_S L_\nu) + \alpha_S g_3(\alpha_S L_\nu) + \ldots \} 
\]

\( L_\nu = \ln \nu \)

The functions \( g_1, g_2, g_3 \) control LL, NLL, NNLL contributions, respectively.

The are defined such that \( g_i(\lambda = 0) = 0 \)

Note: NLL terms formally suppressed by one power of \( \alpha_S \) with respect to LL. The expansion is as systematic as the ordinary perturbative expansion.

Note: The terms coming from \( g_2 \) are of the same order as those coming from the combined effect of \( H^{(1)} \) and \( g_1 \).

At NLL we have to include \( g_1, g_2 \) but also \( H^{(1)} \).
How to combine resummed cross section to fixed order?

matching procedure \[ \hat{\sigma} = \hat{\sigma}_{\text{res.}} + \hat{\sigma}_{\text{fin.}}. \]

Start from resummed contribution \( \hat{\sigma}_{\text{res.}} \) which includes all the logarithmically enhanced terms

Define:

\[ \hat{\sigma}_{\text{fin.}} = \hat{\sigma}_{\text{f.o.}} - [\hat{\sigma}_{\text{res.}}]_{\text{f.o.}}. \]

standard fixed order result

In this way the calculation is everywhere as good as the fixed order result but much better in the region where soft gluon effects are important.
Parton showers

Provide an all-order approximation of the partonic cross section in the soft and collinear regions

- somewhat similar to resummed calculations

+ Much more flexible, since they can give a fully exclusive description of the final state

+ Make possible to include hadronization effects

- Difficult matching with fixed order

- No analytical information

What about logarithmic accuracy?
Resummed calculations can be in principle performed to arbitrary logarithmic accuracy.

The logarithmic accuracy achievable by parton showers is instead limited by quantum mechanics.

Parton showers are essentially probabilistic: quantum interference cannot be taken into account.

This problem is overcome by using colour coherence: soft gluon radiated at large angles destructively interfere.

The effect of quantum interferences is thus approximated by angular ordering constraint.

Angular ordering allows to reach “almost” NLL accuracy (for inclusive enough observables).

The extension to higher logarithmic accuracy is not necessarily feasible.
Resummation: an explicit example

Consider the production of a vector boson at small transverse momentum

Two-scale problem: \( q_T \ll Q = M_Z \)

The recoiling gluon is forced to be either soft or collinear to one of the incoming partons

Real radiation strongly inhibited: KLN cancellation still at work but large logarithmic contributions of the form \( \alpha_S^n \log^2 n \frac{Q^2}{q_T^2} \) appear spoiling the perturbative expansion

\[
\begin{align*}
\text{LO:} & \quad \frac{d\sigma}{dq_T} \rightarrow +\infty \\
\text{as} & \quad q_T \rightarrow 0 \\
\text{NLO:} & \quad \frac{d\sigma}{dq_T} \rightarrow -\infty
\end{align*}
\]
Single-gluon contribution in the small $q_T$ limit

$$\frac{\alpha_S}{2\pi} \int d\omega \frac{d\theta^2}{\theta^2} \left( \delta^2(q_T - q_{T1}) - \delta^2(q_T) \right) C_F \frac{1 + z^2}{1 - z}$$

= \frac{\alpha_S}{2\pi} \int dq_{T1}^2 \frac{q_{T1}^2}{q_{T1}^2} \left( \delta^2(q_T - q_{T1}) - \delta^2(q_T) \right) C_F \int_0^{1 - q_{T1}/Q} dz \left( \frac{2}{1 - z} - (1 + z) \right)

= \frac{\alpha_S}{2\pi} \int_0^{Q^2} dq_{T1}^2 \frac{q_{T1}^2}{q_{T1}^2} \left( \delta^2(q_T - q_{T1}) - \delta^2(q_T) \right) \left( A^{(1)} \log \frac{Q^2}{q_{T1}^2} + B^{(1)} + \ldots \right)

= \frac{\alpha_S}{2\pi} \int \frac{d^2b}{(2\pi)^2} e^{ibq_T} \int_0^{Q^2} \frac{dq_{T1}^2}{q_{T1}^2} \left( e^{-ibq_{T1}} - 1 \right) \left( A^{(1)} \log \frac{Q^2}{q_{T1}^2} + B^{(1)} \right)

Performing irrelevant azimuthal integration

Neglect $O(q_{T1}/Q)$ terms

Universal coefficients

$A^{(1)} = C_F$

$B^{(1)} = -\frac{3}{2} C_F$
More gluons: in b-space kinematics fulfill exact factorization

\[ \delta^2(q_T - q_{T1} - \ldots - q_{Tn}) \rightarrow e^{ib(q_T - q_{T1} - \ldots - q_{Tn})} = e^{ibq_T} \prod_i e^{-ibq_{Ti}} \]

Adding a $1/n!$ symmetry factor the single gluon contribution exponentiates

\[ \int \frac{d^2 b}{(2\pi)^2} e^{ibq_T} \exp \left\{ \int_0^{Q^2} \frac{dq_{T1}^2}{q_{T1}^2} (J_0(bq_{T1}) - 1) \frac{\alpha_S}{2\pi} \left( A^{(1)} \log \frac{Q^2}{q_{T1}^2} + B^{(1)} \right) \right\} \]

Replacing \( \alpha_S \rightarrow \alpha_S(q_{T1}^2) \) we can control subleading effects

The resummed cross section is now finite as \( q_T \rightarrow 0 \)

This is what we observe in the data!

G. Parisi, R. Petronzio (1979)
G. Gurci, M. Greco, Y. Srivastava (1979)
J. Kodaira, L. Trentadue (1982)
An improved b-space formalism

We use b-space resummation and introduce some novel features

G. Bozzi, S. Catani, D. de Florian, MG (2005)

Parton distributions are factorized at $\mu_F \sim M$

$$\frac{d\hat{\sigma}_{ac}^{(\text{res.})}}{dq_T^2} = \frac{1}{2} \int_0^\infty \, db \, b \, J_0(bq_T) \, \mathcal{W}_{ac}(b, M, \hat{s}; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2)$$

$$\mathcal{W}_N(b, M; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2) = \mathcal{H}_N(\alpha_S(\mu_R^2); M^2/\mu_R^2, M^2/\mu_F^2)$$

where the large logs are organized as:

$$\mathcal{G}_N(\alpha_S, bM; M^2/\mu_R^2, M^2/\mu_F^2) = L \, g^{(1)}(\alpha_S L)$$

$$+ g^{(2)}_N(\alpha_S L; M^2/\mu_R^2) + \alpha_S \, g^{(3)}_N(\alpha_S L; M^2/\mu_R^2, M^2/\mu_F^2) + \ldots$$

with $L = \ln M^2 b^2 / b_0^2 \quad \tilde{L} = \ln \left(1 + M^2 b^2 / b_0^2\right) \quad \text{and} \quad \alpha_S = \alpha_S(\mu_R)$

Unitarity constraint enforces correct total cross section
The $q_t$ spectrum of the Higgs

We applied the formalism to compute the Higgs spectrum at the LHC

NLL+LO and NNLL+NLO results with consistent study of theoretical uncertainties and high quality matching to fixed order

Integral of resummed spectra reproduces the correct NLO and NNLO total cross sections

Calculation implemented in the fortran code HqT available at

http://theory.fi.infn.it/grazzini/codes.html
Higgs spectrum: comparison of different approaches

- RESBOS: basically NLL+LO accuracy: NLO at large $q_T$ included though a K-factor
- Berger et al.: basically (N)NLL+LO
- Kulesza et al.: joint resummation of transverse momentum and threshold corrections
- HERWIG 6.3 (no ME correction)
- PYTHIA 6.2 (ME correction included)

Reasonable agreement in shape but Pythia 6.2 considerably softer!
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Nice agreement with MC@NLO

NLO result (not shown) diverges to $+\infty$ as $p_T^{WW} \to 0$

NNLL effect tends to make the spectrum harder

Resummation effects generally small on leptonic observables

Effects seen only when hard cuts are applied
Recent progress: automated resummation

Fixed order NLO calculations typically implemented in observable-independent parton levels MC codes

Resummed calculations usually worked out (when possible) analytically for each observable generally painful

CAESAR

Automatic resummation of a large class of event-shape variables

Avoids the need to find the conjugate space in which the observable factorizes
Price to pay: more limited range of applicability and accuracy

To be done: matching with fixed order
Recent progress: non-global logs

Non global observables: sensitive to emissions in only a part of phase space

These observables are affected by previously neglected single logarithmic contributions

Resummed in closed form only in the large $N_c$ limit

These effects are due to soft-gluon radiation at large angles difficult to take them into account in MC parton shower

Recently: even “superleading” terms discovered in gap-between jets cross section at hadron colliders

M. Dasgupta, G. Salam (2001)

Summary

Resummed calculations allowed us to push the validity of QCD perturbation theory to the boundary of the available phase space where fixed order predictions are not reliable

Resummed predictions are automatically provided by standard MC:

- Much more flexible, since they can give a fully exclusive description of the final state
- Make possible to include hadronization effects
- Difficult matching with fixed order
- Logarithmic accuracy often unclear
- Difficult to estimate uncertainties
Analytical resummations provide the most advanced theoretical accuracies available at present

- Up to NNLL in some cases (threshold, $QT$, EEC)
- Easy matching with fixed order
- Easier to estimate uncertainties
- Have to be worked out for each observable (but progress in automatization is being made)

Bottom line:

MC and analytical resummations are complementary! Analytical resummed calculation will be particularly helpful in the validation of MC simulation tools
EXTRA SLIDES
But try now with cuts used for Higgs search in the $H \rightarrow WW \rightarrow l\nu l\nu$ channel

$\begin{align*}
\pt_{\text{min}} &> 25 \text{ GeV} & 35 \text{ GeV} < \pt_{\text{max}} &< 50 \text{ GeV} \\
\pt_{\text{miss}} &> 20 \text{ GeV} & \Delta\phi &< 45^\circ & m_{ll} &< 35 \text{ GeV} \\
\pt &< 30 \text{ GeV}
\end{align*}$

The $M_{WW}$ distribution is completely off at NLO! The position of the peak is shifted by about 50 GeV.